Approximating Correlated Defaults

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Introduction

- Since 1980s: debut of fixed-income *portfolio* products.
- Useful: allocate risks via securitization.
  - CMOs (prepayment risk); CDOs, CDSs (default risk).
- Allocating these risks reduces borrowing costs for all.
- Must measure portfolio risk; cannot assume independence.
  - Typical portfolio metric: correlation/covariance matrix.
  - However, correlation is linear; default is non-linear.
- Ideally, we would like to:
  - Understand default dependence/clustering; and,
  - Measure portfolio diversification.

Results

- Find approximation for intuitive reduced-form model.
- Approximation is elegant, consistent, and novel.
- Yields idea: default-approximating portfolio of iid bonds/loans.
- This approach also gives us two useful risk metrics:
  - ILC = iid loan count in default-approximating portfolio;
  - IDR = iid default rate of those iid loans.
- Also, this lets us approximate the default-time distribution.

CDOs and CDSs

- Collateralized debt obligations (CDOs) are complicated.
- CDO tranches give priority of allocating defalts to investors.
- Credit default swaps (CDSs), written on bonds/CDOs.
- CDOs and CDSs are just bond/bond portfolio derivatives.
- The difficulty in modeling CDOs/CDSs: they involve defaults.
- Tough because defaults are rare events.
- A CDO tranche (*e.g.* first 5% of defaults) seems harder.
- Turns out some theory handles tranches with ease.

Time to Default

- Think of defaults as involving two pieces:
  - Time to default ($\cong$PD, our concern); and, loss given default.
- Often model waiting times as exponentially-distributed.
  - Like flipping coins every time period.
  - If k heads, a default occurs.

Correlated Defaults: Why?

- However, it seems likely that defaults are related.
- More simply:
  - Do we think defaults stay constant in recessions?
  - Are laid-off coworkers all more likely to default?
- Formally: Borrowers may share certain risk factors.
  - Sensitivity to national, local economy; and,
  - Sensitivity to certain industries, companies.
- Past few years: record losses on portfolio defaults.
- Seems default correlations/dependence were not well-modeled.

Correlated Defaults: Difficult

- Why were these dependence structures ignored?
- Jarrow and Yu (2001) modeled 2 bonds with cross-holding issuers.
  - For more bonds "working out these distributions is more difficult."
- Even harder if we are considering a tranche of a CDO.
- What if we knew each bond's default distribution, dependences?
  - Exact portfolio default distribution is still very difficult.
- How to model dependence?
  - Typically: Copulas. Easy to use vs. opaque, nonlinear mapping.
  - I approximate average portfolio default distribution.

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Approximating Correlated Defaults

- First assume a structural-ish reduced-form model:
  - Events occur when random "alarm clocks" go off[1].
  - Alarms related to systematic (common) risk factors; and,
  - Alarms related to idiosyncratic (borrower-specific) risks.
- The model dynamics can then be thought of as:
  - When idiosyncratic alarm rings, that borrower defaults.
  - When systematic alarm rings, macro event occurs (*e.g.* US recession).
  - Idiosyncratic clocks then speed up for exposed borrowers.
- This allows a statistical approximation (Edgeworth expansion).

[1] The exponential timers we refer to in stochastic processes.

Edgeworth Expansions

- Edgeworth expansion: base distribution plus correction terms.
- Expansions use cumulants (like centered moments).
  - First four cumulants: mean, variance, skewness, kurtosis.
- Cumulants determine base distribution parameters, corrections.
- Typically, the base distribution is the normal distribution.
- Instead, I expand about a gamma distribution:
  - Sum of iid exponential random variables is gamma-distributed.
  - Sum of non-iid, correlated exponential r.v.s?
  - Base gamma distribution implied by cumulants is close.

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Edgeworth Expansion of Gamma

- What do expansions look like? If Y is average default time,

$$\begin{aligned}\hat{f}_Y(y) = \gamma_{\hat{\nu},\hat{\lambda}}(y) &+ \frac{\kappa_3\hat{\lambda}^3 - 2\hat{\nu}}{6}\sum_{i=0}^{3}(-1)^{3-i}\binom{3}{i}\gamma_{\hat{\nu}-i,\hat{\lambda}}(y) \\ &+ \frac{\kappa_4\hat{\lambda}^4 - 6\hat{\nu}}{24}\sum_{i=0}^{4}(-1)^{4-i}\binom{4}{i}\gamma_{\hat{\nu}-i,\hat{\lambda}}(y) \\ &+ \frac{(\kappa_3\hat{\lambda}^3 - 2\hat{\nu})^2}{72}\sum_{i=0}^{6}\binom{6}{i}(-1)^{6-i}\gamma_{\hat{\nu}-i,\hat{\lambda}}(y) \\ &+ O(n^{-3/2})\end{aligned} \tag{1}$$

- Mean, variance, skewness, kurtosis: $\frac{\hat{\nu}}{\hat{\lambda}}, \frac{\hat{\nu}}{\hat{\lambda}^2}, \kappa_3, \kappa_4$.

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Economic Meaning of Parameters

- Edgeworth expansion parameters yield economic insight.
- $\hat{\nu}$ is the iid-equivalent loan count (ILC).
  - Unrelated, equal-size loans needed for similar default risk.
  - "This portfolio defaults like a portfolio of $\hat{\nu}$ iid bonds."
  - Thus ILC $\hat{\nu}$ measures portfolio default-relative diversification.
  - Theoretically-based metric *vs* Moody's KMV "diversity score."
- $\hat{\lambda}$ is the iid-equivalent default rate (IDR).
  - Measures credit quality/default probability of iid loans.
- Pseudocumulants $\left(\frac{\kappa_3\hat{\lambda}^3-2\hat{\nu}}{6}, \frac{\kappa_4\hat{\lambda}^4-6\hat{\nu}}{24}\right) \overset{?}{\Rightarrow}$ default clusters.

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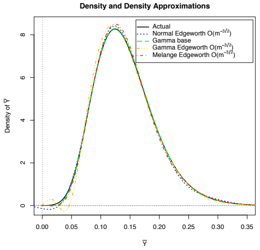
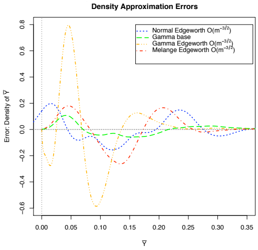
Weaknesses of This Approach

- One problem: We have switched terminology.
- Want: portfolio default times; instead model *average* default times.
- Often use Edgeworth expansions to model non-average distributions.
- Chambers (1967): this is OK if regularity conditions hold.
- Further, the ideas may still translate to the portfolio.
- Caveats:
  - Expansions may yield areas of "negative probability."
  - Portfolio cumulants estimated via censored individual loans.

Simulation: 200-bond CDO Equity Tranche

- Simulate 5% equity tranche of a 200-bond subprime CDO[2].
- Risk factor: US economy. (average 1 event/20 years)
- Mean unaccelerated default times 5–20 years (BBB–BB credit).
- Crisis-accelerated default times 1–4 years (B–CCC credit).
- Use cumulants of equity tranche (first 10) default times.





MSEs:

Normal Edge.: 0.0034

Gamma base: 0.0006

Gamma Edge.: 0.0306

Mèlange Edge.: 0.0051

[2] Exaggerated number of bonds (200 ¿ 125) for illustration.

Simulation: Interpretation

- Can see value of gamma base distribution.
- 10 bonds in equity tranche have ILC $\hat{\nu} = 7.8$ bonds.
- Those 7.8 bonds would have IDR $\hat{\lambda}$ =6/year.
- Thus mean time to default $\approx$ 2 months (C credit).
- Tranche distribution implied by approximating portfolio:
  - Tranche life $\sim Exp(\hat{\lambda}/\hat{\nu})$.
  - Mean tranche life: 15.6 months.

General: Modeling Ease

- We can easily observe prior (realized) default times.
- Can also use estimates of macro risk event frequencies.
- May then use survival analysis, maximum likelihood:
    - Allows us to estimate unobserved cumulants.
    - Estimating these cumulants is a much simpler problem.
- Estimated cumulants imply average default distribution.
- That helps us assess value/risk of CDO tranches, CDSs.
- I give an example of this in the paper.

Conclusion

- Found distribution approximation for mean bond default time.
  - Leads to an elegant (novel?) Edgeworth expansion.
  - Consistent for structural model of interacting "alarms."
- Approximating portfolio parameters also have economic meaning:
  - $\hat{\nu}$: ILC $=$ iid loan count in approximating portfolio.
  - $\hat{\lambda}$: IDR $=$ iid loan default rate in approximating portfolio.
- May be used to imply default distribution for tranche/portfolio.