Market Structure, Counterparty Risk, and Systemic Risk

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Counterparty Risk

- **Counterparty**: other side of ongoing financial agreement.
  - A bank enters into a swap with you on the S&P 500.

- **Counterparty Risk**
  - Risk resulting from default/bankruptcy of a counterparty.
  - Strictly: Risk to you from one of your counterparties.
  - Broadly: Includes effects on overall market (our concern).

- This broad definition we refer to as *systemic risk*. 
Counterparty Risk to Systemic Risk

- Counterparty risk affects market when large failure looms:
  - Near-bankruptcy of Bear Stearns (May 2008)
  - Bankruptcy of Lehman Brothers (Sep 2008)
  - Bankruptcy of Refco Inc? (Oct 2005, owned #1 CME broker)

- Outstanding notional at CME before ceasing trading:
  
<table>
<thead>
<tr>
<th>Counterparty</th>
<th>Outstanding Notional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bear Stearns</td>
<td>$761 BB</td>
</tr>
<tr>
<td>Lehman Brothers</td>
<td>$1,150 BB</td>
</tr>
<tr>
<td>Refco LLC</td>
<td>$130 BB</td>
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</tbody>
</table>

- N.B. No defaults or trade halts at CME for these events.


- Is counterparty risk an “accelerant” in financial crises?
Distress increases volatility sharply and significantly.

- Widens spreads: transactions costs $\uparrow$; market liquidity $\downarrow$.
- Volatility is pushed onto the survivors (externality).

Crisis bankruptcies have real costs:

- Virtuous, vicious circles of market and funding liquidity\(^2\).
- Reduced funding liquidity affects non-financial firms also.
- Less invested in risky assets; allocative inefficiency?
- Higher unemployment: harder job searches, lower tax revenue.
- Bernanke (1983): affects credit markets; possible depression.

\(^2\)Brunnemeier and Pedersen (2009).
Market structure affects contagion and exposure to defaults.

Specifically: complete networks magnify systemic risk.
  - Difference due to differing creation of complete networks.
  - Also: financial, banking networks differ (cf Acemoglu).

Market fragility estimable with a few metrics of market core.

Can price distress volatility of differing structures.
Model: Market Structures

- Investigate two extremes of \( n \)-counterparty networks.

```
1   2
|\  | \|
+---+---
|   |   |
+---+---
     CCP
```

Star network  
(Market with CCP\(^3\))

```
1 ——— 2
|   |   |
+---+---
|   |   |
+---+---
     3 ——— 4
```

Complete network  
(Bilateral “OTC” market)

- Each node is a counterparty (capital \( K \), risk aversion \( \lambda \)).
- Each edge is a contract\(^4\) linking counterparties \( i \) and \( j \).
- Contract exposure: \( q_{ij} = -q_{ji}; \ q_{i<j} \overset{iid}{\sim} N(0, \eta^2) \)
- Counterparty \( i \)'s net exposure: \( Q_i = \sum_{j \neq i} q_{ij} \).
- Same net exposures (\( Q_i \)'s) in both networks.

\(^3\)Central counterparty.
\(^4\)A swap or forward on a risky asset.
To study counterparty risk, events occur at discrete times.

$t = 0$: Bankruptcy of counterparty $n$ occurs.
- All contracts with counterparty $n$ are invalidated.
- Pushes unwanted exposure onto other $n - 1$ counterparties.

$t = 1$: Living counterparties trade in response to bankruptcy.

$t = 2$: Living counterparties close out bankruptcy-induced exposure.

Order of trading in a period is random, not strategic.
Model: Price Impact of Trading

- Each counterparty $i$ trades $x_i$ shares at time $t = 1$.
  - Impact has linear permanent component$^5$.
  - Permanent component impacts prices for later traders.
- Trade ordering, price impact create low and high prices.
- Time periods are very short; two simplifying assumptions:
  1. Prices have no drift other than price impact due to trading.
  2. Price diffusion is Gaussian (not log-normal).
- Defer handling crisis-related adverse selection.

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$^5$Price impact could arise from inventory risk cost, non-crisis adverse selection.
Effects of Invalidated Contracts

- Suppose counterparty A is net long the market.
- $\Rightarrow$ Other counterparties are net short the market.
- These are their preferred equilibrium positions.
- Thus when counterparty A defaults:
  - Survivors must re-create exposure from counterparty A.
  - Survivors become net sellers.
- CCP market: only CCP trades; net sell.
- OTC market: some counterparties will sell, some will buy.
- However, counterparties trade in own interest.
  - Do they rehedge immediately? Push market further?
Consider bankruptcy of a large financial firm.
Assume large market move $r_0$ at $t = 0$ induces bankruptcy.
Net exposure $Q_n$ probably large; estimate via EVT$^6$.

\[
\hat{Q}_n = \frac{-K}{r_0} + \frac{\eta \sqrt{n-1}}{c_n(1 - e^{-e^{-c_n\kappa_1-d_n}})} \sum_{k=1}^{\infty} (-1)^{k+1} e^{-k(c_n\kappa_1+d_n)} \frac{k^{k+1}}{kk!}
\]

where $\kappa_1 = \frac{-K}{r_0\eta\sqrt{n-1}}$ (minimum exposure causing death),
$c_n = \frac{1}{\sqrt{2 \log(n)}}$, and $d_n = \sqrt{2 \log(n)} - \frac{\log \log(n) + \log(16 \tan^{-1}(1))}{2 \sqrt{2 \log(n)}}$.

$^6$Equivalent: endow all counterparties with perfect information, examine most likely $Q_n|r_0$. 
Large Bankruptcies

- For large $Q_n$, trading at $t = 1, 2$ will move market a lot.
- Move will be further in direction that caused bankruptcy.
- This raises two distressing possibilities:
  - Contagion: move may cause other counterparties to fail; or,
  - Checkmate: hedging may bankrupt the hedger.
- Counterparties anticipate these, respond selfishly.
- For bilateral OTC market, all counterparties may trade.
  - All hedge anticipated follow-on bankruptcy exposure $\hat{Q}_f$.
  - Trouble: $\nu > 1$ (overtrading at $t = 1$) to be expected.
  - Longs, shorts may largely self-segregate rehedge timing.
- Thus network structure matters.
CCP anticipates follow-on bankruptcies; equilibrium yields

Follow-on bankruptcy exposure $\hat{Q}_f$ (distress exposure):

$$\hat{Q}_f = (n - 1)^{3/2} \eta \frac{\phi(\kappa_2) - \phi(\kappa_1)}{\Phi(\kappa_1)}$$

where

$$\kappa_2 = \frac{-Kp_0}{\eta \sqrt{n - 1}}$$

and $p_0 r_0 - \pi (\hat{Q}_n + \hat{Q}_f)$ = min exposure for follow-on death.

Follow-on bankruptcies $\hat{b}$ (distress pervasiveness):

$$\hat{b} = (n - 1) \frac{\int_{\kappa_2}^{\kappa_1} \phi(z) dz}{\int_{-\infty}^{\kappa_1} \phi(z) dz} = (n - 1) \left( 1 - \frac{\Phi(\kappa_2)}{\Phi(\kappa_1)} \right)$$
Large Bankruptcy: Equilibrium OTC Net Trade

- OTC traders anticipate one another, follow-on bankruptcies.
- However: those most at-risk rehedge quickly, others delay.
- Random trade sequence ⇒ uncertain low of rehedging $S_{n-1}$.
- Use these to solve for equilibrium OTC net trade.

$$\kappa_2 = \frac{-Kp_0}{\eta \sqrt{n-1} (p_0 r_0 + \pi E(S_{n-1} | \nu))},$$  \hspace{1cm} (4)

$$\hat{Q}_f = (n-1)^{3/2} \eta \frac{\phi(\kappa_2) - \phi(\kappa_1)}{\Phi(\kappa_1)}.$$  \hspace{1cm} (5)

- Important to note that $\nu \geq 1$ (in $E(S_{n-1})$).
- Finding $\nu$ is hard: $n$-player (random) game; usually c1.75.
Bad Behavior? Checkmate and Hunting

Proposition (Checkmate)

A large enough initial bankruptcy may yield a follow-on bankruptcy in expectation — despite any finite effort by the troubled counterparty.

Proposition (Hunting)

For a complete network of 3 or more counterparties and a large enough initial bankruptcy, two or more other counterparties may profit by driving a survivor into (follow-on) bankruptcy.
The Other Extreme: A Separating Equilibrium?

- Another (extreme) possibility exists in bilateral OTC markets:
  - Buyers and sellers may separate when they trade.
  - Those who are same side as net rehedge rush to hedge first.
  - Those on other side wait to allow maximum distress.
- If net rehedge makes sellers panic, net sale in period 1 is:
  \[-E\left(\sum_{i=1}^{n-1} [x_i] - \sum_{i=1}^{n-1} x_i\right) = -\hat{Q}_n - \hat{Q}_f\] (6)
  \[-n - 1)^{3/2} \eta \phi(\mu^*) - (\hat{Q}_n + \hat{Q}_f)(1 - \Phi(\mu^*))\] (7)
  where \(\mu^* = \frac{\hat{Q}_n + \hat{Q}_f}{(n-1)^{3/2}\eta}\) (net rehedge in std devs/survivor)
  and \(\phi, \Phi\) are standard normal pdf, cdf.
Consider large bankruptcy for $n = 10$ counterparties.$^7$

Std deviation of bilateral contract exposure $\eta = 1,000,000$.

Distress exposure $\hat{Q}_f$ and pervasiveness $\hat{b}$ vs. $\hat{Q}_n$.

Lines: (P)ooled OTC; (S)eparated OTC; (C)CP

$P - S$: Envelopes of distress exposure, pervasiveness

$^7$Price impact parameters are as in Almgren and Chriss (2001).
Large Bankruptcies: Example of Market Impact

- Suppose $\hat{Q}_n = 10,000,000$; GARCH variance decay of 0.9.
- For CCP market:
  - Expected market impact: $-\$30$.
  - Effective annual volatility goes from 30% to 38%.
- If pooled OTC buyers, sellers overtrade $1.75 \times$ at $t = 1$.
  - Expected market impact: $-\$31$.
  - Annual volatility ↑ to 328% (instant.), 146% (effective).
- If OTC buyers and sellers separate, at $t = 1$:
  - Expected market impact: $-\$41$.
  - Annual volatility ↑ to 596% (instant.), 268% (effective).
Large Bankruptcies: Example of Real Effects

- Suppose $\hat{Q}_n = 10$ MM, market size of $40$ MM$^8$.
- If 8% equity premium and mean risk aversion of $\hat{\lambda} = 3$:
  - Equilibrium allocation to risky asset: 29% (71% cash).
  - Post-crisis: 19% (CCP), 1.2% (OTC pool), 0.4% (OTC sep).
- Cost of distress externality:
  - $3.2$MM (CCP), $123$ MM (OTC pool), $425$ MM (OTC sep).
  - Cost of OTC market distress is $3–11 \times$ market size.
- Given 2–3 bankruptcies; mean employees, compensation:
  - 260,000–400,000 unemployed; $33–$49 billion pay loss.
  - At 40% total taxes: revenue loss of $13–$20 billion.
- Also affects credit markets, overall macroeconomy.

$^8$Approximately $2(\hat{Q}_n + \hat{Q}_f)$. 
Large Bankruptcies: Not So Random

- Complete networks admit two destabilizing events:
  - Checkmate: weak counterparty may have no beneficial trade.
  - Hunting: counterparties force others into bankruptcy.
- Worse, hunting is a full equilibrium behavior.
  - Market may be pushed far beyond one follow-on bankruptcy.
- Are counterparties selfishly amoral/evil? Maybe not.
  - Trade amount may pre-hedge expected follow-on bankruptcies.
  - This reduces surprise need for trading in period 2.
- CCP markets have fewer such destabilizing events.
  - Suggests central clearing reduces OTC market volatility.
Difference from Allen and Gale (2000)

- Allen and Gale (2000): complete networks are more robust.
- I disagree: complete networks are more fragile.
- Allen and Gale approach: top-down.
  - Net exposure: $Q_i \sim N(0, (n-1)\eta^2)$
  - Contract exposure: $q_{ij} = Q_i/(n-1)$. (all same sign)
- My approach: bottom-up.
  - Contract exposure: $q_{i<j} \sim N(0, \eta^2); q_{ij} = -q_{ji}$;
  - Net exposure: $Q_i = \sum_{j \neq i} q_{ij}; \; Q_i \sim N(0, (n-1)\eta^2)$.
- Same net exposures $Q_i$'s, different contract exposures $q_{ij}$'s.
- Strategic separation of buyers, sellers unlikely in A&G.
Conclusion

- Even small bankruptcies temporarily increase volatility.
- For a large bankruptcy in a bilateral OTC market:
  - Counterparties may be unable to save themselves (checkmate).
  - Counterparties may hunt their weakest peers for profit.
  - Volatility externality (and thus cost) higher than CCP market.
- Self-segregating buyers, sellers in OTC markets can be nasty:
  - Externality distress cost $\gg$ market size. (market failure?)
- Suggests benefits to centralized clearing in OTC markets$^9$.
- Volatility externality cost $\Rightarrow$ when to move markets to CCP.
- May be able to measure when markets are more/less brittle.
  - $n, \eta, \bar{K}$ for part of market like complete network.

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$^9$Biais, Heider, Hoerova (2011) suggests CCP is capital efficient.