Index Arbitrage and Refresh Time Bias in Covariance Estimation

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Variance and Covariance Estimation

Classical problem with many financial applications:

- Risk management: VaR, ES, risk budgeting;
- Portfolio optimization and asset allocation;
- Option valuation and hedging;
- Market making: inventory risk;
- Pairs trading/relative value strategies;
- Forecasting, rate of information flow, “liquidity.”
- Estimate corporate bond variances, default probabilities.
- Covariance risk: time-varying betas, Sharpe ratios.

High-Frequency Data

Estimation increasingly done with high-frequency data. Allows:

- Study intraday pattern of volatility/covariance;
- Improve volatility/covariance forecasts;
- Portfolio performance gain worth 50–200 bp annually[1].
- Time-varying correlations, variances, betas, Sharpe ratios;
- High-frequency estimates crucial for market making, HFT;
- Post IPO/merger: quicker estimation allows more investment.

[1] Fleming, Kirby, Ostdiek (2003)

Variance, Covariance Literature

Much work on high-frequency variance and covariance estimation:

- Handling microstructure noise/asynchronicity
  1. Kernel-based approach:
     Barndorff-Nielsen, Hansen, Lunde, Shephard (2008, 2010)
  2. Pre-averaging: Podolskij, Vetter (2009);
     Christensen, Kinnebrock, Podolskij (2010)
  3. Two-scales Realized Variance, Covariance:
     Zhang, Mykland, Aït-Sahalia (2005); Zhang (2010)
- Handling jumps
  1. Bipower Variation, Covariation:
     Barndorff-Nielsen, Shephard (2004a, 2004b)
  2. Median Realized Volatility:
     Andersen, Dobrev, Schaumburg (2008)

Index Arbitrage

- *Index Arbitrage*: Trade index members vs. futures/ETF.
  - Simple application of APT; has been done for decades;
  - Increasing automation greatly eases trading.
- US indexes: Dow 30, Nasdaq 100, S&P 500, Russell 2000 (!).
- Myth: Too expensive/fussy to trade all those stocks. (Why?)
- *Spread*: $\delta_t = \underbrace{\sum_{i=1}^{N} w_i S_{it}}_{\text{Index}} - \underbrace{F_t}_{\text{Futures}}$
- Strategy: Trade stocks vs. futures/ETF when $|\delta|$ "large."

Index Arbitrage Bias

- Index arb pushes index, futures, ETF toward each other.
- Worse: trades determine (mostly) contemporaneous returns.
    - Index arbitrage creates simultaneous index members trades.
    - Index arbitrage often create trades when $|\delta_t|$ large.
- Thus price co-movement is due to *two* DGPs:
    - Similarity of economic fundamentals (Σ);
    - Reversion (O-U?) of index-ETF-futures prices (δ).
- Spread δ_t biases estimates of variance, covariance.
- We suspect the bias is larger for illiquid stocks.

Classical Model for Financial Data

Often assume geometric Brownian motion:
Let X_t be a vector of log-stock prices $\log(S_t)$ at time t;

$$dX_t = \underbrace{\mu dt}_{\text{drifts}} + \underbrace{\Sigma dW_t}_{\text{diffusions}} . \quad (1)$$

We also can augment this to address inadequacies:

- Stochastic volatility;
- Leverage effects;
- Account for microstructure noise; and,
- Incorporate jumps.

Classical Model with Index Arbitrage

Index arbitrage adds an O-U term to the standard drift+diffusion:

$$dX_t = \underbrace{\mu dt}_{\text{drifts}} + \underbrace{\Sigma dW_t}_{\text{diffusions}} - \underbrace{\gamma \delta_t / S_t}_{\substack{\text{index arb}\\ \text{effects}}} \tag{2}$$

$$d\delta_t = \lambda(\delta_t^* - \delta_t) + \sigma_\delta dZ_t \tag{3}$$

where

γ = price sensitivities to spread δ_t, $\gamma > 0$, $\gamma \overset{?}{\propto} w/2$; and,

λ = speed of mean reversion.

Index Arbitrage and Variance Estimation

- Spread δ_t biases estimates of variances σ_i^2.
- Continuous-time bias is easy to determine:

$$\mathrm{Var}(dX_{it}) = \sigma_i^2 + \gamma_i^2 \frac{\sigma_\delta^2}{2\lambda S_{it}^2}; \quad (4)$$

$$E(\hat{\sigma}_i^2) > \sigma_i^2. \quad (5)$$

- We often "sample" by computing returns between trades.
- But index arb causes trades $\Rightarrow$ not sampling at random.
- More trades if $|\delta_t|$ large $\Rightarrow$ larger observed effect.
  - Endogeneity in trade times *and* spread return sizes[2].

[2] Thus Li, Mykland, Renault, Zhang, Zheng (2011 WP) does not hold.

Index Arbitrage and Covariance Estimation

- Spread δ_t also biases estimated covariances $\hat{\Sigma}_{ij}$.
- Continuous-time bias is easy to determine:

$$\mathrm{Cov}(dX_{it}, dX_{jt}) = \Sigma_{ij} + \gamma_i \gamma_j \frac{\sigma_\delta^2}{2\lambda S_{it} S_{jt}}; \tag{6}$$

$$E(\hat{\Sigma}_{ij}) > \Sigma_{ij}. \tag{7}$$

- Recall: index arb causes trades $\Rightarrow$ not random sampling.
- However, bias may be worse for covariance estimation.
  - Covariance estimation must handle asynchronous trading.
  - Most covariance estimates use "refresh times;" but,
  - Refresh times amplify over-sampling of index arb comovement.

Refresh Times and Asynchronous Trading

We use refresh times to handle asynchrony of trading. However:

- Many non-index-arb trades do not create refresh times.
- Index arbitrage trades create refresh times.
  - ⇒ Refresh times discard few/no index arb trades.
- Thus over-sampling (and bias) likely worse than for variance.

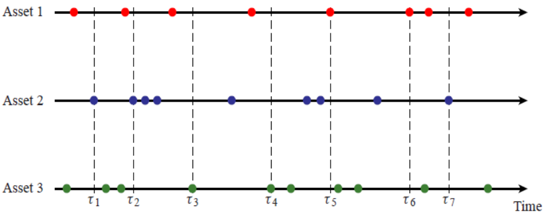


Figure 1: Example refresh times. Source: Barndorff-Nielsen *et al.* (2010)

Data

- Look at some data to see if we find these effects.
- Index: Dow Jones Industrial Average (DJIA)
- ETF: S&P depository receipt, (DIA) $=$ DJIA/100 $\pm c$
- Sample period: 1–31 October 2008.
- Data source: NYSE Trade and Quote (TAQ) Database.
- Data cleaning as in Barndorff-Nielsen *et al.* (2009).

Index Arbitrage Spread Construction

- Construct tick-by-tick DJIA bid and ask prices.
- To avoid exchange clock differences, compare to DIA ETF.
- *Spread*: $\delta_t = \underbrace{\sum_{i=1}^{N} w_i S_{it}}_{\text{Index}} - \underbrace{S_{DIA,t} \times 100}_{\text{ETF}}$
- Note more trading when spread is large: over-sampling.

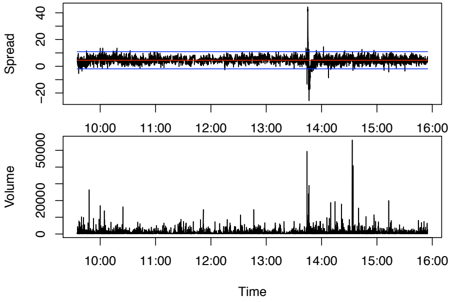


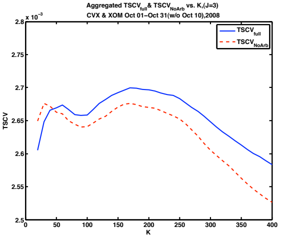
Figure 2: Dow 30 index-ETF spread on 1 Oct 2008

Index Arb Covariance Bias: Cleaning

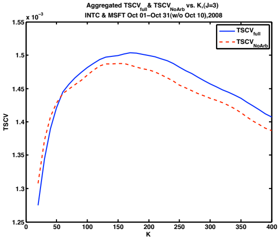
- To see if index arb refresh times have an effect, remove them.
- Flag trades when spread $\delta_t > 2$ s.d.s from daily mean.
- Compute TSCV with and without flagged trades.
- Allow slow time scale to vary to see limiting behavior.
- Same-sector pairs all show overestimation of covariance.

Index Arb Covariance Bias: Plots

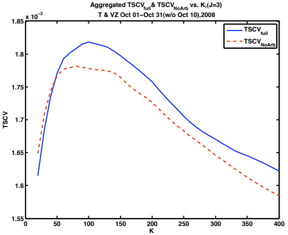
Same-sector pairs all show overestimation of covariance.



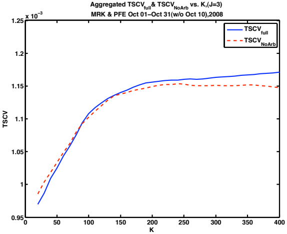
ChevronTexaco vs. ExxonMobil



Intel vs. Microsoft



T-Mobile vs. Verizon



Merck vs. Pfizer

Conclusion

- Shown index arbitrage biases variance, covariance estimates.
- Biases all high-frequency variance, covariance estimation.
- Reasons why refresh times can exacerbate this problem.
- Data analysis: some covariances overestimated by about 3%.
- Overestimated covariances may cause over-diversification.
  - Seems innocuous, but this can raise investors costs.
- Combined overestimates reduce allocations to risky assets.
- Data analysis remains to be done for variance bias.
- Suggests more careful data cleaning needed.