Index Arbitrage and Refresh Time Bias in Covariance Estimation

Dale W.R. Rosenthal    Jin Zhang

University of Illinois at Chicago

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Variance and Covariance Estimation

Classical problem with many financial applications:

- Risk management: VaR, ES, risk budgeting;
- Portfolio optimization and asset allocation;
- Option valuation and hedging;
- Market making: inventory risk;
- Pairs trading/relative value strategies;
- Forecasting, rate of information flow, “liquidity.”
- Estimate corporate bond variances, default probabilities.
- Covariance risk: time-varying betas, Sharpe ratios.
High-Frequency Data

Estimation increasingly done with high-frequency data. Allows:

- Study intraday pattern of volatility/covariance;
- Improve volatility/covariance forecasts;
- Portfolio performance gain worth 50–200 bp annually\(^1\);
- Time-varying correlations, variances, betas, Sharpe ratios;
- High-frequency estimates crucial for market making, HFT;
- Post IPO/merger: quicker estimation allows more investment.

\(^1\)Fleming, Kirby, Ostdiek (2003)
Much work on high-frequency variance and covariance estimation:

- **Handling microstructure noise/asynchronicity**
  1. Kernel-based approach:
  2. Pre-averaging: Podolskij, Vetter (2009);
     Christensen, Kinnebrock, Podolskij (2010)
  3. Two-scales Realized Variance, Covariance:
     Zhang, Mykland, Aït-Sahalia (2005); Zhang (2010)

- **Handling jumps**
  1. Bipower Variation, Covariation:
     Barndorff-Nielsen, Shephard (2004a, 2004b)
  2. Median Realized Volatility:
     Andersen, Dobrev, Schaumburg (2008)
Index Arbitrage

- **Index Arbitrage**: Trade index members vs. futures/ETF.
  - Simple application of APT; has been done for decades;
  - Increasing automation greatly eases trading.
- US indexes: Dow 30, Nasdaq 100, S&P 500, Russell 2000 (!).
- Myth: Too expensive/fussy to trade all those stocks. (Why?)

- **Spread**: \( \delta_t = \sum_{i=1}^{N} w_i S_{it} - F_t \)
  - Index
  - Futures
- Strategy: Trade stocks vs. futures/ETF when \(|\delta|\) “large.”
Index Arbitrage Bias

- Index arb pushes index, futures, ETF toward each other.
- Worse: trades determine (mostly) contemporaneous returns.
  - Index arbitrage creates simultaneous index members trades.
  - Index arbitrage often create trades when $|\delta_t|$ large.
- Thus price co-movement is due to two DGPs:
  - Similarity of economic fundamentals ($\Sigma$);
  - Reversion (O-U?) of index-ETF-futures prices ($\delta$).
- Spread $\delta_t$ biases estimates of variance, covariance.
- We suspect the bias is larger for illiquid stocks.
**Classical Model for Financial Data**

Often assume geometric Brownian motion:
Let $X_t$ be a vector of log-stock prices $\log(S_t)$ at time $t$;

$$dX_t = \mu dt + \sum dW_t.$$  \hspace{1cm} (1)

We also can augment this to address inadequacies:

- Stochastic volatility;
- Leverage effects;
- Account for microstructure noise; and,
- Incorporate jumps.
Index arbitrage adds an O-U term to the standard drift+diffusion:

\[
dX_t = \mu dt + \sum dW_t - \frac{\gamma \delta_t}{S_t} \\
\]

\[
d\delta_t = \lambda (\delta^* - \delta_t) + \sigma_{\delta} dZ_t
\]

where

\(
\gamma = \text{price sensitivities to spread } \delta_t, \ \gamma > 0, \ \gamma \propto w/2; \ \text{and,}\n\)

\(
\lambda = \text{speed of mean reversion.}\n\)
Index Arbitrage and Variance Estimation

- Spread $\delta_t$ biases estimates of variances $\sigma_i^2$.
- Continuous-time bias is easy to determine:

$$\text{Var}(dX_{it}) = \sigma_i^2 + \gamma_i^2 \frac{\sigma_{\delta}^2}{2\lambda S_{it}^2};$$

$$E(\hat{\sigma}_i^2) > \sigma_i^2.$$  \hspace{1cm} (4)  \hspace{1cm} (5)

- We often “sample” by computing returns between trades.
- But index arb causes trades $\Rightarrow$ not sampling at random.
- More trades if $|\delta_t|$ large $\Rightarrow$ larger observed effect.
  - Endogeneity in trade times and spread return sizes$^2$.

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$^2$Thus Li, Mykland, Renault, Zhang, Zheng (2011 WP) does not hold.
Index Arbitrage and Covariance Estimation

- Spread $\delta_t$ also biases estimated covariances $\hat{\Sigma}_{ij}$.
- Continuous-time bias is easy to determine:

$$\text{Cov}(dX_{it}, dX_{jt}) = \Sigma_{ij} + \gamma_i \gamma_j \frac{\sigma^2}{2\lambda S_{it} S_{jt}};$$  \hspace{1cm} (6)

$$E(\hat{\Sigma}_{ij}) > \Sigma_{ij}. \hspace{1cm} (7)$$

- Recall: index arb causes trades $\Rightarrow$ not random sampling.
- However, bias may be worse for covariance estimation.
  - Covariance estimation must handle asynchronous trading.
  - Most covariance estimates use “refresh times;” but,
  - Refresh times amplify over-sampling of index arb comovement.
Refresh Times and Asynchronous Trading

We use refresh times to handle asynchrony of trading. However:

- Many non-index-arb trades do not create refresh times.
- Index arbitrage trades create refresh times.
  - ⇒ Refresh times discard few/no index arb trades.
- Thus over-sampling (and bias) likely worse than for variance.

Figure 1: Example refresh times. Source: Barndorff-Nielsen et al. (2010)
Data

- Look at some data to see if we find these effects.
- Index: Dow Jones Industrial Average (DJIA)
- ETF: S&P depository receipt, (DIA) = DJIA/100 \( \pm c \)
- Data source: NYSE Trade and Quote (TAQ) Database.
- Data cleaning as in Barndorff-Nielsen et al. (2009).
Index Arbitrage Spread Construction

- Construct tick-by-tick DJIA bid and ask prices.
- To avoid exchange clock differences, compare to DIA ETF.
- **Spread**: \( \delta_t = \sum_{i=1}^{N} w_i S_{it} - S_{DIA,t} \times 100 \)
- Note more trading when spread is large: over-sampling.

**Figure 2**: Dow 30 index-ETF spread on 1 Oct 2008
To see if index arb refresh times have an effect, remove them.
Flag trades when spread $\delta_t > 2$ s.d.s from daily mean.
Compute TSCV with and without flagged trades.
Allow slow time scale to vary to see limiting behavior.
Same-sector pairs all show overestimation of covariance.
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ChevronTexaco vs. ExxonMobil
Intel vs. Microsoft
T-Mobile vs. Verizon
Merck vs. Pfizer
Conclusion

- Shown index arbitrage biases variance, covariance estimates.
- Biases all high-frequency variance, covariance estimation.
- Reasons why refresh times can exacerbate this problem.
- Data analysis: some covariances overestimated by about 3%.
- Overestimated covariances may cause over-diversification.
  - Seems innocuous, but this can raise investors costs.
- Combined overestimates reduce allocations to risky assets.
- Data analysis remains to be done for variance bias.
- Suggests more careful data cleaning needed.